Limit with a composite Sine.

https://www.linkedin.com/feed/update/urn:li:activity:6518458725869723648

Calculate the limit

$$\lim_{x\to 0}\frac{\sin(x\sin x)+\sin(x\sin(x\sin x))}{x\sin(\sin x)+\sin(\sin(x\sin x))}.$$

Solution by Arkady Alt , **San Jose** ,**California**, **USA**.

$$\lim_{x\to 0}\frac{\sin(x\sin x)+\sin(x\sin(x\sin x))}{x\sin(\sin x)+\sin(\sin(x\sin x))}=\lim_{x\to 0}\frac{1+\frac{\sin(x\sin(x\sin x))}{\sin(x\sin x)}}{\frac{x\sin(\sin x)}{\sin(x\sin x)}+\frac{\sin(\sin(x\sin x))}{\sin(x\sin x)}}=\frac{1}{2}$$

Indeed, since $\lim_{x\to 0}\sin x=\lim_{x\to 0}\sin(x\sin x)=\lim_{x\to 0}x\sin(x\sin x)=0$ then applying remarkable limit $\lim_{t\to 0}\frac{\sin t}{t}=1$ we obtain

$$\lim_{x\to 0}\frac{\sin(x\sin(x\sin x))}{\sin(x\sin x)}=\lim_{x\to 0}\frac{\sin(x\sin(x\sin x))}{x\sin(x\sin x)}\cdot x=1\cdot 0=0,$$

$$\lim_{x\to 0}\frac{x\sin(\sin x)}{\sin(x\sin x)}=\lim_{x\to 0}\frac{x\sin x}{\sin(x\sin x)}\cdot\frac{\sin(\sin x)}{\sin x}=\lim_{x\to 0}\frac{x\sin x}{\sin(x\sin x)}\cdot\lim_{x\to 0}\frac{\sin(\sin x)}{\sin x}=1\cdot 1=1$$

and $\lim_{x\to 0}\frac{\sin(\sin(x\sin x))}{\sin(x\sin x)}=1.$